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Common Ownership Equilibrium Existence and Its Properties

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April 18, 2017

Abstract

This paper studies an equilibrium model of competition between firms under partial common ownership. Shareholders choose how to vote for managers, who compete for votes by proposing alternative product market strategies. Firms interact as in a Cournot model with differentiated goods. Shareholders have heterogeneous portfolios, and some may hold shares in competing firms. We show that (i) an equilibrium exists, (ii) is unique under certain conditions, and (iii) can be estimated numerically regardless of initial guess. The model suggests a measure of common ownership concentration, PHHI, that avoids some of the pitfalls of the often-used MHHI delta measure and retains benefits, such as nesting the HHI measure of market concentration in the absence of common ownership. In particular, PHHI treats all shareholders with identical portfolios (and thus competitive preferences) as a single shareholder. This feature makes PHHI unsusceptible to manipulation by redistribution of shares among minority shareholders that would change MHHI, yet without a change in their competitive preferences, and therefore has desirable properties for regulatory agencies.

*I thank my advisors David Miller and Martin Schmalz; graduate students Robert Kahn, Eric Wilson, Benjamin Lipsius, and participants of Econ 695/696 presentations.

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1 Introduction

The rise of common ownership among publicly traded U.S. firms is a well-recognized phenomenon in the literature (Azar (2017); Azar et al. (2017); Anton et al. (2016)). The assumption of price-taking firms is crucial for the Fisher separation theorem which ensures that all shareholders unanimously agree on profit maximization objective of the firm (Milne (1974)). When firms interact, the idea of profit maximization as a firm's unanimous objective needs to be abandoned. The search of alternative firm's objectives is justified when firms have some diversified (whose objective is portfolio profit maximization) and undiversified (whose objective is firm-level profit maximization) owners.

The goal of our work is to propose a parsimonious model of competition among firms under partial common ownership within the industry that does not assume firms to be profit maximizers. The model shows how common ownership may lead to reduced competition and it creates the framework for a new measure of market concentration. This new measure is able to aggregate investors with similar/identical portfolios which is not the case for existing measures.

Using this model we plan to establish the existence of equilibrium under certain assumptions, describe the level of competition in an economy as a function of the level of common ownership, evaluate effects of anti-trust measures, propose a new measure of concentration and compare its properties with those of existing measures. The research question can be formulated as: what are the properties and existence conditions of an equilibrium in a common ownership setting?

The model is a one-period game that features firms governed by strategic managers whose objective is to gain the support of at least 50% votes of shareholders (investors) of a given firm. Shareholders follow a well-defined sequence of steps aimed towards maximizing their portfolios' profit given the assumption of conditional sincerity. It includes N firms, $2N$ managers (manager plus contender per firm), and I investors. Investors own portfolios of firms' shares, together they own the entire industry. The goal of an investor is to maximize his portfolio profit at the end of the period. Firms choose the levels of output (set by the managers) and receive individual prices as in the Cournot model with differentiated goods. Investors are not strategic players in this model but managers and contenders are. Each firm has a manager whose only objective is to retain his place. This essentially means that he has to propose a policy that will attract at least 50% of the votes of shareholders when compared to any other possible policy (in a case when the election is initiated by the contender). Contender is a strategic player who has the objective to win a competition against an incumbent manager of the firm (his policy has to attract more than 50% of the votes). We can interpret him as an activist. He proposes policies for the firm and if that policy differs from incumbent manager's proposal an election is held. That means that the incumbent manager effectively competes with a potential manager in choosing the policy to implement in Downsian competition (Downs (1957)). Each investor has his own view on the optimal policy for every firm. Since the model assumes that the distribution of shares is publicly known and that investors are conditionally sincere, the best response for the manager is to choose the weighted-median policy to be implemented. To guarantee the existence of equilibrium I'm using the assumption of continuity for shareholders' optimal policy functions and for managers' best-response functions (Arrow & Hahn (1971)).

Our model proceeds as follows:

1. Investors are endowed with the shares in firms β_{in}, $0 \leq \beta_{in} \leq 1$ and $\sum_{i=1}^{I} \beta_{in} = 1 \; \forall n$. Each investor wants to maximize his portfolio profit $\pi_i = \sum_{n=1}^{N} p_n q_n \beta_{in}$ at the end of the period; where p_n is the price of output for firm n and q_n is the quantity of output for firm n. The production is costless and there are no fixed costs in the model. Investors (shareholders) are not strategic players.

2. Contenders and managers of the firms play a simultaneous game by proposing firms' production decisions q_n. They are strategic players.

3. The prices for the products are determined as in Cournot model with differentiated goods using the following equation:

$$p = Aq + v$$

where p is a $N \times 1$ price vector, q is a $N \times 1$ vector of firms' decisions q_n, v is a $N \times 1$ vector of firms' individual shares of monopolistic power, and A is a $N \times N$ matrix of externalities. Natural assumptions are that the elements of v are positive, the diagonal elements of A are negative, and the off-diagonal elements are non-positive (in this model we do not consider complementary goods for the purpose of simplicity).

4. Manager gets utility 1 if his proposal is a Condorcet winner in competition with any other feasible proposal when shareholders of the firm vote; otherwise manager gets 0.

5. Contender gets utility 1 if his proposal is supported by more than 50 % of shareholders in comparison to manager's proposal, otherwise contender gets 0.

6. Shareholders follow the conditional sincerity assumption: they vote for the candidate truthfully according to their first order optimality condition for the policy of the firm given the policies of other firms being fixed. Their utility functions are single-peaked and symmetric in this case, so each shareholder votes for the manager whose policy is the nearest to his optimal value.

In an equilibrium, each incumbent manager is able to get at least 50% of shareholders' votes against any other policy. That is each manager chooses a policy which is a Condorcet winner in competition with any other possible policy proposal by the Contender. Since the Contender needs strictly more than 50% of the votes to win, the only possibility for him is a mistake of a manager (i.e. manager does not choose a Condorcet winner policy). Hence, without loss of generality, we can consider equilibrium where Contender chooses the same policy as manager.

The conditional sincerity assumption mentioned above is widely used in the literature in different forms. For instance, Azar (2017) uses this assumption directly, DeMarzo (1993) implicitly assumes the same by saying that his definition of majority rule production equilibrium implies "production-taking" behavior on the part of shareholders.

The effects of common ownership on firm's objective function have been discussed in the literature. For instance, Hart (1977) studies the properties of equilibrium under uncertainty in a stock market economy. He shows that take-over bids do not lead to net market value maximization behavior by firms. Benninga & Muller (1979) study the existence of equilibrium under uncertainty and show that an equilibrium when all firms choose production plans approved by a majority of their shareholders exists. Hansen & Lott (1996) study how product market imperfections and portfolio diversification by investors lead to rejection of value maximization as a corporate policy. Davis (2008) documents the rise of concentration of ownership in the hands of a few financial institutions in the early 2000s.

Another relevant piece of literature is DeMarzo (1993). He considers the case of incomplete markets in a two-period model. In the first period the state is known, but in the second period there are S possible states and N assets where $S > N$. Hence investors may disagree on the state prices and consequently on their voting decisions. Note that in this model the investors' voting decisions effectively shape the space of available assets and each investor would like to make that shape to serve his own utility function. This is not related directly to the profit maximization problem as there is a disagreement over the state prices. Also, the decisions of the firms in this model do not explicitly affect the output of the other firms. The effect can be only translated through the changes in voting behavior in response to changes in the shape of space of available assets. DeMarzo (1993) also talks about corner cases where the dimensionality of decisions' space for each firm is 1 ($S_j = 1$) or when the number of states of the world is higher than the number of assets available by 1 ($S - N = 1$). Note that these cases are still in the framework of incomplete markets, and even though they have the similar solution concept they do not cover the model we consider in our work.

Azar (2017) shows that common ownership among publicly traded firms is now well present. For instance, the probability that two random S&P 1500 firms have a common shareholder with at least 5% stakes in both firms is around 90% in 2014Q4. Also, he develops a partial equilibrium oligopoly model with shareholders voting for the manager to run the firm given that the firm's policy will affect profits of other companies that might be in their portfolios. His model adopts the assumption that managers can not perfectly predict the voting behavior of shareholders due to a random term in their utility functions. The probabilistic approach is one of the key components that shapes the firm's objective in this case. It also has its imprint in the solution: the firms' objective functions depend on the assumed distributions of the random utility terms. The paper does not suggest any particular distribution assumptions for these random terms, but to draw any intuition from the suggested expressions for the objectives of the firms the reader has to do it himself.

The model we propose follows Azar (2017) closely, but it differs in the assumption on utility functions of investors. We assume that utility functions of investors are simply their payoffs at the end of the period, i.e. there is no random component. The managers and contenders are perfectly aware of shares' allocation, externalities of every firm, and the utility functions of investors, hence the voting procedure is deterministic in our case. It significantly changes the solution concept since the absence of uncertainty in voting procedure makes the voting function for a particular manager candidate non-differentiable, and hence the usual first-order condition approach has no bite in this model. This difficulty pays off by removing the need to specify the distributional assumptions on the random component of utility function which may hinder the understanding of the objectives of a firm. Azar (2017) mentions the case of deterministic voting but does not include it into his model.

A measure of market concentration is an important tool in anti-trust policy. Current literature provides different measures of market concentration. For instance, Modified Herfindahl-Hirschman Index (MHHI) that takes into account the effects of common ownership (Bresnahan & Salop (1986); Salop & O'Brien (2000)). MHHI arises from a solution of optimization problem in Cournot model with homogeneous goods for the weighted sum of firm's shareholders' profit and has the following formula:

$$MHHI = \sum_k \sum_j \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}} \right) s_k s_j = HHI + \sum_k \sum_{j \neq k} \left(\frac{\sum_i \gamma_{ij} \beta_{ik}}{\sum_i \gamma_{ij} \beta_{ij}} \right) s_k s_j$$

where s_k is the market share of firm k; β_{ik} is the ownership share of investor i in firm k; γ_{ij} is the measure of the degree of control over firm j by owner i; and $HHI = \sum_k s_k^2$. Our contribution to the literature is the introduction of a new measure of market concentration inspired by the equilibrium properties of the model studied in our paper. We propose the following measure of market concentration that accounts for investors' preferences and aggregates investors with similar ones:

$$Projection\ HHI = \sum_{k=1}^{N} \sum_{j=1}^{N} \left(\sum_{i=1}^{I} \sum_{l=1}^{I} \frac{\langle \beta_{i\cdot}, \beta_{l\cdot} \rangle}{\sqrt{\langle \beta_{i\cdot}, \beta_{i\cdot} \rangle \langle \beta_{l\cdot}, \beta_{l\cdot} \rangle}} \gamma_{ik} \gamma_{lj} \right) s_k s_j$$

where $\langle \beta_{i\cdot}, \beta_{l\cdot} \rangle$ is a scalar product of ownership vectors. This formula has several useful properties. First, in the simple case without common ownership, it gives result identical to the HHI (as MHHI does). Second, in a case with common ownership it can be represented in two parts: HHI and the correction responsible for common ownership (MHHI does that as well). Third, in a case when two investors have identical portfolios (up to a scale factor) this measure treats them as a single investor with combined portfolio (MHHI does not) which is consistent with equilibrium properties of our model. Yet this measure comes not without flaws. It is designed to be compatible with the HHI when there are no diversified investors in the industry. That makes the formula harder to justify by using only the equilibrium conditions of the model discussed.

Overall, we see the contribution in proposing a model where firm's objective is set by a manager who balances the interests of shareholders to retain his place; showing that equilibrium exists in the case of common ownership; describing the properties of the equilibrium; and proposing a new measure of market concentration.

2 Model

The model has one period. All actions take place in the beginning of the period and then profit realization happens. The model consists of three main pieces: firms, managers + contenders, and investors. These pieces are related as following.

The modeled industry contains N firms. Here we do not model entry/exit decisions. Each firm decides on production parameter q_n (e.g. output in case of Cournot or price in case of Bertrand) and its profit is determined in an interaction with decisions of other firms. Firms in this model are not strategic players.

The decision on behalf of each firm is done by the manager of this firm by submitting a proposal b_n. Managers are strategic players. The goal of a manager is to convince 50% or more of the shareholders (investors of the firm) that his decision is the best possible for them. We motivate this goal as if the manager wants to retain the support of shareholders to keep his place in the firm and not to be replaced with a potential manager (contender) who may have a proposal that has wider support. Managers (including contenders) do not cooperate/communicate with each other and play a simultaneous game.

There are I investors in the industry. Investors have utility functions $U_i(\pi_i)$ which are strictly increasing in wealth at the end of the period π_i (realized profit of the portfolios). There is a numeraire in the model, but it is only used to evaluate the profits of the firms. Investors are not endowed with numeraire in the beginning of the period, instead, they are endowed with the shares in the firms $1 \geq \beta_{in} \geq 0$. Note that all investors together own all firms in the industry (i.e. for any firm n we have $\sum_{i=1}^{I} \beta_{in} = 1$). The realized profit is then divided according to the shares that investors own in each firm. Investors are not strategic players and they are conditionally sincere. Each investor has the following protocol to maximize his utility. The incumbent manager is opposed by a potential manager (contender) who would like to take his place by proposing a policy with wider support. In case these proposals are different an election is initiated. Then investors in this firm solve the optimization problem given the policies of other firms being fixed and vote in a bilateral competition for the manager which proposes the better policy for them. We assume that one share equals one vote, but this assumption may be easily relaxed. The assumption of conditional sincerity allows us to say that investors vote truthfully.

The game is defined in the following way. The set of players consists of N managers (a manager per firm) and N potential managers (contenders). The strategy space for every manager and every contender is $A_n = [0, Q]$ where Q is the maximum possible output for a firm. We think of Q as a finite number to simplify the proof of equilibrium existence. Since further we assume that each firm has a negative externality on itself the finiteness of Q does not restrict us in any way. Payoff function for a manager of a firm is 1 when his action in the game is supported by at least 50% of votes of shareholders in a non-strategic voting procedure. Otherwise he gets 0. Contender gets 1 if his proposal is supported by more than 50% of votes of shareholders and he gets 0 otherwise.

The structure of the shareholdings (β_{in} for all i, n) is publicly known. Managers and contenders play a simultaneous game in which they determine the policy parameter of each firm. In equilibrium managers submit policy proposals that are Condorcet winners in comparison with any other possible policy proposal. Hence the best response for the contenders is any feasible policy proposal. To simplify the discussion we concentrate on the subset of equilibria when contenders also submit the Condorcet-winning proposals but they do not win the tie.

The important features of the model are the continuity of optimal choices of investors wrt firms' production choices and the upper hemi-continuity of managers' decisions wrt optimal choices of investors. These are also accompanied with the idea that in reasonable setting the feasible set of firms' production decisions is bounded and closed (i.e. compact).

2.1 Cournot case

For the sake of simplicity, we are going to adopt the linear Cournot model as an example that leads to continuity in investors' proposals. Since the firms are not strategic players the Cournot model is implemented as follows. The prices that firms get for their products are defined by a matrix equation:

$$p = Aq + v$$

where p is a $N \times 1$ price vector, q is a $N \times 1$ vector of firms' decisions q_n, v is a $N \times 1$ vector of firms individual share of monopolistic power, and A is a $N \times N$ matrix of externalities. Natural assumptions are that the elements of v are positive, the diagonal elements of A are negative (the more of a good firm produces the lower the price is), and the off-diagonal elements are non-positive (in this model we do not consider complementary goods for the purpose of simplicity). The firm gets profit equal to $p_n q_n$ and distributes it to investors according to their shares. Hence the profit of an investor at the end of the period is $\pi_i = \sum_{n=1}^{N} p_n q_n \beta_{in}$.

To see the optimal firm's choice for an investor, consider two specific cases. First, suppose that each firm is fully owned by a separate investor (and hence we have a perfect competition). Then this investor will vote for a manager that chooses the output which will maximize the profit of the firm $p_n q_n$. Consider the following solution for a firm:

$$\max_{q_n} p_n q_n = \max_{q_n} [Aq + v]_n q_n =$$

$$= a_{n1} q_1 q_n + a_{n2} q_2 q_n + ... + a_{nn} q_n^2 + ... + a_{nN} q_N q_n + v_n q_n.$$

The first derivative wrt q_n gives us the FOC:

$$a_{n1}q_1 + a_{n2}q_2 + .. + 2a_{nn}q_n + ... + a_{nN}q_N + v_n = 0$$

hence the optimal level of output in the competitive case is

$$q_n = -\frac{v_n}{2a_{nn}} - \sum_{k \neq n} \frac{a_{nk}q_k}{2a_{nn}}.$$

Now we can generalize this solution and represent it as a linear problem. Construct matrix B as $[B]_{nn} = 2a_{nn}$ and $[B]_{nk} = a_{nk}, n \neq k$, then

$$q = -B^{-1}v$$

is the optimal solution in the competitive case for all firms. The investors vote for the managers that implement these proposals. Since for each firm there is only one investor with 100% shares the equilibrium in the simultaneous game with managers' decisions is trivial: each manager implements the best policy for this single shareholder.

Second, suppose now that all the firms in the economy are owned by a single investor. He would like to maximize his profit at the end of the first period, so the optimization problem would look like

$$\max_{q_1,\dots,q_N} \pi_1 = \max_{q_1,\dots,q_N} \sum_{n=1}^{N} p_n q_n = \max_{q_1,\dots,q_N} q^T(Aq + v) =$$

$$= \max_{q_1,\dots,q_N} p_1q_1 + p_2q_2 + ... + p_Nq_N =$$

$$= \max_{q_1,\dots,q_N} \left(\sum_{n=1}^{N} a_{1k}q_k + v_1 \right) q_1 + \left(\sum_{n=1}^{N} a_{2k}q_k + v_2 \right) q_2 + ... + \left(\sum_{n=1}^{N} a_{Nk}q_k + v_N \right) q_N$$

hence the first order condition for an arbitrary firm k is

$$v_k + 2a_{kk}q_k + \sum_{n \neq k} a_{nk}q_n + \sum_{n \neq k} a_{kn}q_n = 0.$$

The matrix representation of this solution can be constructed in the following form: $[C]_{nn} = 2a_{nn}$ and $[C]_{nk} = a_{nk} + a_{kn}, n \neq k$, then

$$q = -C^{-1}v$$

is the solution in the case of monopoly. It is easy to see that solution, in this case, has almost the same form as in the competitive case. The difference comes only from the off-diagonal terms of the matrix C in comparison to matrix B. As we will see below, the partial common ownership case lies in between of these two.

The problem is that in the case of common ownership shareholders may be diversified enough so for a given company there is no shareholder with more than 50% weight of votes. Before we will resolve this problem we are going to solve the optimal decision problem for a diversified investor. The profit for this investor is $\pi_i = \sum_{n=1}^{N} p_n q_n \beta_{in}$. Then the problem of diversified investor for an arbitrary firm k in is

$$\max_{q_k} \pi_i = \max_{q_k} \sum_{n=1}^{N} p_n q_n \beta_{in} = \max_{q_k} \sum_{n=1}^{N} \beta_{in} \left(\sum_{k=1}^{N} a_{nk} q_k + v_n \right) q_n.$$

The first order condition is

$$\beta_{ik} \left(v_k + \sum_{l \neq k} a_{kl} q_l + 2a_{kk} q_k \right) + \sum_{l \neq k} \beta_{il} a_{lk} q_l = 0.$$

Hence the optimal solution for the entire industry from the perspective of investor i can be represented in matrix form in the following way: $[D]_{nn} = 2\beta_{in} a_{nn}$ and $[D]_{nk} = \beta_{in} a_{nk} + \beta_{ik} a_{kn}, n \neq k$

$$q = -D^{-1} diag(\beta_{i1}, \beta_{i2}, ..., \beta_{iN}) v.$$

Note that this solution is a generalization of two previous cases, but not a direct one. The reader should not think of the matrix equation above as just a solution to a system of linear equations that describes an equilibrium. This is true for the case of perfect monopoly (only 1 investor), but for any other case this is just a view of the investor i on the optimal firms' decisions. In the equilibrium, it is possible that not all (if any) of the firms' decisions (q_n) will coincide with the investor's view on optimal firms' choices.

In our model investors are not strategic players and they use the following procedure that is intended to maximize their utility at the end of the period. For an arbitrary firm k an investor i can come up with the optimal decision that comes from the first order condition above

$$b_{ki} = \frac{1}{2a_{kk}} \left(-v_k - \sum_{l \neq k} a_{kl} q_l - \frac{1}{\beta_{ik}} \sum_{l \neq k} \beta_{il} a_{lk} q_l \right)$$

when $\beta_{ik} > 0$ and $b_{ki} = 0$ when $\beta_{ik} = 0$. This optimal decision is then used by the investor in voting for the preferred manager of the firm k. If voting procedure is initiated for the firm k then this investor in any bilateral comparison of two managers will vote for the manager that is closer to b_{ki}. Though the investor is not strategic, his actions still can be justified partially due to the linearity of the first order condition with respect to firm's choice of output q_k. Hence we can see that the output decision q_k that is further away from his view of optimal policy b_{ki} delivers him higher deviation of first order condition's LHS from zero and the investor in this model will always prefer the least possible deviation of first order condition from zero (optimality) to maximize the profit of his portfolio.

Here we have to revert to a small but important remark. We assume that investor comes up with optimal output values on a firm-by-firm basis (i.e. he solves individual equations for each firm separately taking the output of other firms as given). We call this assumption conditional sincerity of investors. In this way, the set of equations that determine equilibrium is composed of the equations of shareholders that are pivotal in corresponding firms. For instance, in monopoly case, this assumption has no effect since all equations come from a single investor hence the equilibrium solution will coincide with the optimum solution for that investor. But in the case of perfect competition, all equations come from different investors and the equilibrium solution differs from the optimal solutions for each individual investor (each investor wants only his firm to function to derive higher profit). The case of partial common ownership is perhaps the most complicated case here since now investors also interact at a firm level and the space of actions of investors is essentially much larger now. But for the sake of tractability of the model, we would like to restrict interactions between the investors (i.e. no transfers allowed) and rule out complicated strategies (e.g. when voting is not aligned with the optimal choices from first order conditions). Partially this can be justified by the idea that investors may have bounded rationality.

The assumption that prices depend linearly on quantities produced is a good first order approach to describe real world yet it is not perfect. We would like to impose an extra assumption on this optimization problem. It is reasonable to think that the production process is irreversible and hence the optimal quantities from investors' perspectives above do not always lie in a feasible region of parameter space. Hence we would like to impose assumption that

$$b_{ki} \geq 0, \forall n, i$$

which also give us $q_n \geq 0, \forall n$. This assumption also helps us to satisfy compactness requirement of the Kakutani theorem. We leave a more elaborate discussion on continuity and compactness in the Appendix.

Recall that both managers and contenders are strategic players. They simultaneously make proposals for optimal firms' policies. Then shareholders vote for the best proposal as following. We say that proposal of incumbent manager g is preferred by the shareholders of the firm n over proposal d of a potential manager if

$$\sum_{i=1}^{I} \mathbb{I}[g \succsim d]\beta_{in} \geq 0.5$$

i.e. there are at least 50 % shareholders that would prefer g over d. In this case the manager will not get fired from the firm after the end of the period (even though there is no future in this game we assume that manager would like to retain his place). This definition also implies that there might be a region of implementable policies instead of a unique one. This happens when the investors are divided equally between two rival proposals. Then we say that any policy in between is implementable in this firm. Also, the set of implementable policies is always convex. See the discussion on upper hemi continuity of implementable proposals in the Appendix.

Now, given the notation covered above, we can update our definition of the game to make it more rigorous.

- **Players:** Game has $2N$ players: N firms' managers and N potential managers that compete with them.
- **Strategies:** Each player has strategy space $A_n = [0, Q]$ where Q is a finite upper bound for feasible production choices of the firms.
- **Payoffs:** For firm n let g_n be the proposal of the incumbent manager and d_n be the proposal of the contender. Then the payoff function for the incumbent manager of the firm n is

$$\mathbb{I}\left[\sum_{i=1}^{I} \mathbb{I}[g_n \succsim d_n]\beta_{in} \geq 0.5\right]$$

where $\mathbb{I}$ is an indicator function. And the payoff function for the contender at firm n is

$$\mathbb{I}\left[\sum_{i=1}^{I} \mathbb{I}[g_n \succsim d_n]\beta_{in} < 0.5\right].$$

The allocation of the shares is publicly known in this game. Players do not communicate/coordinate with each other and play a one-period simultaneous game. The proof of equilibrium existence is presented in the Appendix.

3 Equilibrium properties

3.1 Local convergence

The model is a one-period game which does not describe the process of reaching the equilibrium. Yet the equilibrium can be described as a solution of a linear system of equations which arises from the preferences of the median shareholders when ties are absent. The equations rely upon the definition of the voting mechanism used by shareholders. In this paper we use the conditional sincerity assumption that eliminates significant technical hurdles. It allows us to think that every shareholder is a non-strategic player who comes up with a voting decision for each firm separately (given the outputs of other firms being fixed) and truthfully (he votes for the manager with a better proposal for him). In a one-period game we interpret the fixed outputs of other firms as the equilibrium outputs.

It is important to know what happens to the equilibrium in case of involuntary deviation from it. For instance, a firm suffers an unexpected delay in its production chain and produces less output than it meant to produce. The simple one-period model concludes that this is an off-equilibrium situation, and we cannot say much about it. To gain more insights we can modify the model in the following way. Suppose there is a sequence of periods $t = 1, 2, 3, ...$, and in every period a modified one-period game is played. In these modified games shareholders, when making voting decisions, treat equilibrium outputs from preceding game as being fixed to decide on optimal output for each particular firm. This differs from our regular one-period game where they look at current equilibrium. Since shareholders are not strategic players, this change does not interfere with the idea of Nash-equilibrium: strategies of managers and contenders remain the same as before. To start the sequence of games, we assume that at period $t = 0$ the vector of firms' outputs q is determined exogenously.

For an arbitrary period, let q' be the vector of equilibrium outputs, and q be the vector of outputs in the previous period. Also, let m be a $N \times 1$ vector of shareholders that are pivotal in corresponding firms (i.e. k-th element of this vector, m_k, describes the pivotal shareholder in k-th firm). Then using the optimal decision condition derived above we can define the new equilibrium as

$$q_k^{'} = \frac{1}{2a_{kk}} \left(-v_k - \sum_{l \neq k} a_{kl} q_l - \frac{1}{\beta_{m_k k}} \sum_{l \neq k} \beta_{m_k l} a_{lk} q_l \right)$$

or

$$q' = A + Bq,$$

so q' is the new value of output that managers propose given the old value of output q. Matrices A and B are defined as following:

$$A = \begin{pmatrix} -\frac{v_1}{2a_{11}} \\ -\frac{v_2}{2a_{22}} \\ \dots \\ -\frac{v_N}{2a_{NN}} \end{pmatrix} \qquad B_{ij} = \begin{cases} 0 & \text{for} \quad i = j \\ -\frac{1}{2a_{ii}} \left(a_{ij} + \frac{\beta_{m_i j}}{\beta_{m_i i}} a_{ji} \right) & \text{for} \quad i \neq j \end{cases}.$$

Let an isolated equilibrium be an equilibrium with a point in the output space which for some $\epsilon > 0$ has an ϵ-ball within which vector of pivotal shareholders m remains the same. In non-modified one-period game equilibrium with output point $\hat{q}$ has the following property: $\hat{q} = A + B\hat{q}$. In case the sequence of equilibria's outputs converges in the output space, the result of it is $q' = q$, so the same property ($q = A + Bq$) holds in the limit. Suppose that previous period ($t - 1 > 0$) equilibrium is isolated, and its output is σ away from the equilibrium output $\hat{q}$ in non-modified one-period game (but σ is small enough for vector m to remain the same), then $q' = \hat{q} + \sigma' = A + B(\hat{q} + \sigma)$ and $\sigma' = B\sigma$. Note that if all eigenvalues of B lie within the unit circle then $||B\sigma||_2 < ||\sigma||_2$ (i.e. B is a contracting linear operator). Hence for an isolated equilibrium, this property of matrix B enables the local convergence property of equilibria's output sequence towards the equilibrium output in a non-modified one-period game since any initial deviation σ in output converges to zero. This also implies local uniqueness of equilibrium in non-modified one-period game.

In a case when equilibrium is not isolated (i.e. the output point is situated on a boundary between two or more regions of output space with different sets of pivotal shareholders) the condition above transforms into several conditions: one per each region that contains equilibrium output point (here we assume that boundaries are included into regions and overlap).

When ties are present (i.e. when two equal 50% coalitions can be formed) the local convergence needs a bit more complicated requirement due to ill-defined vector m. Since two (or more) versions of this vector are possible (i.e. when m_k has two possible values for the firm k with a tie) we need to check both versions of matrix B built using this vector. And in this case convergence means that a deviation will converge to an equilibrium, but it does not need to be the same since we may have a continuity of equilibria. Local uniqueness does not apply here.

3.2 Uniqueness

The discussion on local convergence of equilibrium provides sufficient condition for local uniqueness, but it does not say anything about global uniqueness. In this section, we discuss conditions required and show the global uniqueness of equilibrium in a case without ties. To simplify our discussion assume that the model does not have a corner solution equilibrium (i.e. all firms produce strictly positive amounts of output in equilibrium). Later this assumption can be relaxed. Recall that production decision space is $[0, Q]^N$ where Q is some large quantity that no firm would like to have output above Q. For every point in this production decision space and every firm the model describes a set of pivotal shareholders. If this set is a singleton for every firm, then this point of production space is surrounded by an ϵ-ball of points with the same vector m of pivotal shareholders. The entire production space, excluding its boundaries, can be partitioned up to I^N regions with different vectors of pivotal shareholders. On the boundary between two different regions vector m is not well defined since for at least one firm there exist at least two shareholders with exact same views that happen to be pivotal in this firm. Each such region is a closed set (i.e. includes its boundary). Note that due to continuity of optimal choices of investors and continuity of managers best responses (recall that ties are excluded) give us the continuity of composite function which maps points of production space into itself.

Suppose now that each region with its own vector of pivotal shareholders satisfies the local stability requirement described in the previous section. Then only one interior equilibrium may exist in the model.

To prove this statement consider the following situation. Suppose that two interior equilibria q_a and q_b exist. Then we have $q_a = A_a + B_a q_a$ and $q_b = A_b + B_b q_b$ where matrices A and B are defined in the previous section for corresponding regions of each equilibrium. This means that these two equilibria are fixed points and the Euclidian distance between q_a and q_b in the production decision space is unchanged by making the recursive step towards an equilibrium (convergence to the one-period game's equilibrium in a sequence of games). Now consider a straight line connecting q_a and q_b. Since the production decision space is convex this line entirely lies within the space and may cross several regions with different vectors m in between, see fig. 1 for a schematic representation. Now consider the sum of Euclidian distances along the path from q_a to q_b. Since the path follows a straight line this sum is equal to the distance from q_a to q_b, i.e.

$$||q_a - q_b||_2 = ||q_a - q_1||_2 + \sum_{k=1}^{\text{last step}-1} ||q_k - q_{k+1}||_2 + ||q_{\text{last step}} - q_b||_2.$$

Then at the next iteration (next period) we have $||A_a + B_a q_a - (A_b + B_b q_b)||_2 = ||q_a - q_b||_2$ and $||A_k + B_k q_k - (A_k + B_k q_{k+1})||_2 = ||B_k(q_k - q_{k+1})||_2 < ||q_k - q_{k+1}||_2$. Last equality uses the fact that boundaries are parts of each region and q_k lie

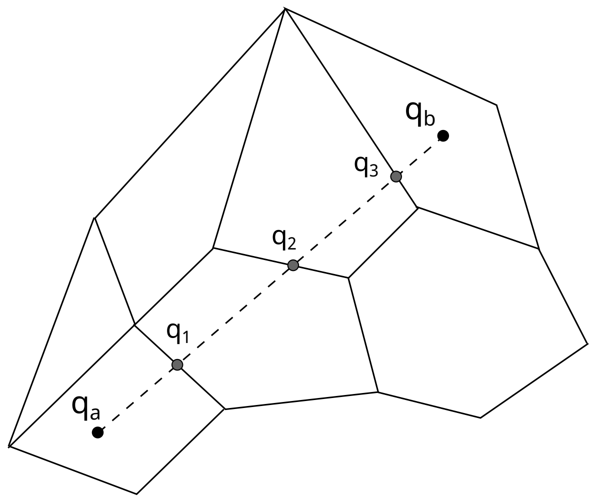

Figure 1: Schematic diagram of two equilibria in the production decision space separated by several regions with different sets of pivotal investors

at the boundaries. This implies

$$||q_a - q_b||_2 > ||B_a(q_a - q_1)||_2 + \sum_{k=1}^{\text{last step}-1} ||B_k(q_k - q_{k+1})||_2 + ||B_b(q_{\text{last step}} - q_b)||_2$$

which along with the continuity of mapping function $(A + Bq)$ implies that Euclidian distance between q_a and q_b is strictly larger than sum of Euclidian distances along some continuous path from q_a to q_b. Since this is not possible we can conclude that two interior equilibria can not exist in this model simultaneously given aforementioned conditions.

3.3 Global convergence

The uniqueness result in the previous section gives us a hint for establishing a global convergence result for equilibrium. When we impose all conditions necessary for uniqueness of the equilibrium we also automatically get global convergence towards the equilibrium of a one-period game in a sequence of modified games. To see this we need to follow the proof of uniqueness. Suppose now that q_a is the equilibrium point and q_{b0} is some point far away from q_a.

Then as before we can construct a straight line between q_a and q_{b0} that will shrink to a continuous path between q_a and q_{b1} after applying a step into the next period in the sequence of modified games (q_{b0} transforms into q_{b1}). As we know from the proof above, the length of that path is smaller than initial length of a straight line between q_a and q_{b0}, hence the length of a straight line between q_a and q_{b1} can not exceed the path length. This let us conclude that q_{b1} lies strictly closer to q_a than q_{b0}.

This result is particularly important from computational point of view. As numbers of firms and investors get bigger the time required to calculate a solution grows at faster than exponential rate. That makes the applicability of the model proposed quite limited (8×8 case is probably the ceiling for a desktop computer and reasonable time allotment). The global convergence result justifies the numerical convergence approach which has significantly smaller computational difficulty.

4 PHHI

The level of competition in the market is associated with the market power of participants. Basic Cournot model suggests that market concentration is partially responsible for the presence of market power among larger firms. This makes measures of market concentration particularly relevant in antitrust literature. Bikker & Haaf (2002) conclude that Herfindahl-Hirschman Index (HHI) along with k-bank concentration ratio are used most frequently both in theory and practice in research of banking markets. Yet both of these do not grasp the influence of partial common ownership on competition. Bresnahan & Salop (1986) and Salop & O'Brien (2000) propose a Modified HHI which arises from a solution of the optimization problem in Cournot model with homogeneous goods. They use the assumptions that goods are homogeneous and each firm has an objective to maximize the weighted sum of firm's shareholders' profit. The index has the following form:

$$MHHI = \sum_k \sum_j \left(\frac{\sum_i \gamma_{ij}\beta_{ik}}{\sum_i \gamma_{ij}\beta_{ij}} \right) s_k s_j = HHI + \sum_k \sum_{j \neq k} \left(\frac{\sum_i \gamma_{ij}\beta_{ik}}{\sum_i \gamma_{ij}\beta_{ij}} \right) s_k s_j$$

where s_k is the market share of firm k; β_{ik} is the ownership share of investor i in firm k; γ_{ij} is the measure of the degree of control over firm j by owner i; and $HHI = \sum_k s_k^2$. Azar et al. (2017) find a significant effect of common ownership on ticket prices in the US airline industry. They show that $\Delta MHHI = MHHI - HHI$ has a significant positive effect on average fares.

Yet MHHI has its own drawbacks. Consider the following numerical example.

$$s = \begin{pmatrix} 0.3 \\ 0.3 \\ 0.3 \\ 0.1 \end{pmatrix} \quad \beta_A = \begin{pmatrix} 0.05 & 0.1 & 0.15 & 0 \\ 0.05 & 0.1 & 0.15 & 0 \\ 0.45 & 0.4 & 0.35 & 0 \\ 0.45 & 0.4 & 0.35 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \beta_B = \begin{pmatrix} 0.1 & 0.2 & 0.3 & 0 \\ 0.45 & 0.4 & 0.35 & 0 \\ 0.45 & 0.4 & 0.35 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This example has 4 firms with market shares defined by vector s and ownership structure defined by matrix β_A. We also assume that voting rights correspond the cash flow rights ($\gamma = \beta$). This gives us $MHHI(s, \beta_A) = 0.8123$. Suppose now that two smaller investors are going to merge and ownership structure now described by β_B. The new $MHHI(s, \beta_B) = 0.8013$ is smaller than its value before the merger. At the same time, we can see that ownership structure β_A admits higher number of coalitions of investors that deliver at least 50% of voting power in all 3 firms than ownership structure β_B. The model discussed in this paper delivers identical equilibria under both ownership structures due to perfect alignment of joining investors. Hence the model suggests that concentration of ownership essentially remains the same. One can argue that combined investor has higher bargaining power due to larger absolute shareholdings, but this reasoning fails for a slightly modified example.

$$\beta_C = \begin{pmatrix} 0.05 & 0.1 & 0.15 & 0 \\ 0.05 & 0.1 & 0.15 & 0 \\ 0.9 & 0.8 & 0.7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \beta_D = \begin{pmatrix} 0.1 & 0.2 & 0.3 & 0 \\ 0.9 & 0.8 & 0.7 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Then $MHHI(s, \beta_D) - MHHI(s, \beta_C) = -0.0073$ is again a decrease in concentration measure but even combined investor has no chance to negotiate the firms' policies. It is reasonable to expect that in this case the measure of market concentration should remain the same. Also we get that $MHHI(s, \beta_A) = MHHI(s, \beta_D)$ which is aligned with the model presented in this paper but does not fit the logic of previous examples. Consider now a third example which shows how MHHI can be exploited to comply with DOJ/FTC guidelines by slightly changing the ownership structure in a merger. Suppose that an industry has 3 firms and 4 investors with market shares $s = [0.4, 0.4, 0.2]$ and the ownership structure is defined below.

Current, MHHI: 5987 Desired, MHHI: 6216 Feasible, MHHI: 6184

$$\begin{pmatrix} 0.1 & 0.49 & 0 \\ 0.02 & 0.11 & 0 \\ 0.88 & 0.4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0.1 & 0.49 & 0 \\ 0.0 & 0.0 & 0 \\ 0.9 & 0.51 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{pmatrix} 0.1 & 0.49 & 0 \\ 0.027 & 0.0153 & 0 \\ 0.873 & 0.4947 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The third investor would like to acquire the portfolio of the second investor to achieve the desired ownership structure. This merger leads to a change in MHHI by 229 points. DOJ/FTC 2010 Horizontal Merger Guidelines suggest that changes of HHI over 200 are "presumed likely to enhance market power". In this case, the burden to prove that the merger does not lead to adverse anti-competitive effects is on the merging parties. To avoid dealing with all these complications the third investor may consider another ownership structure where after the merger he puts apart about 3% of his total assets. For instance, he can trade with the second investor. This trade allows him to reduce the change in MHHI below the 200 points threshold.

Note that in this case the portfolios of second and third investors now are the same up to a scaling factor. These portfolios are also the same as the portfolio of the third investor after a merger in the desired case. This means that they all have the identical incentives regarding firms' strategies. The model we present in this paper yields the same equilibrium for both desired and feasible ownership structures. The concentration measure presented in this chapter aggregates such investors and assigns identical scores to ownership structures after the merger in this example.

These examples show that interpretation of MHHI in case of a merger is difficult. The advantage of MHHI is that it can be easily analytically derived from Cournot model with homogeneous goods under the assumption that each firm maximizes the weighted sum of portfolios of its shareholders. The examples above are made to question this assumption. They show that some shareholders may be large enough to dictate the optimal policies for firms completely ignoring any effects on minority shareholders. MHHI does not properly account for that.

Geroski (1983) suggests that construction of concentration indices subject to data constraints usually involves regressing potential candidates on an aggregate measure of industry performance (e.g. market share weighted sum of price-cost margins). We will use the similar approach to construct a measure that is compatible with HHI and uses the same data as MHHI. Blackorby et al. (1982) provide desirable properties of number-equivalents (i.e. using equivalent number of equal-sized firms as a measure of concentration), and the merger property requires such indices to decrease or remain constant if two firms merge and receive their pre-merger combined total output. We can adopt this property and apply it towards a merger of investors in our model in the following way: the measure of market concentration should increase or remain the same if two investors merge and combined total output remains the same or falls.

Let's introduce a new index for market concentration: Projection HHI.

$$Projection\ HHI = \sum_{k=1}^{N}\sum_{j=1}^{N}\left(\sum_{i=1}^{I}\sum_{l=1}^{I}\frac{\langle\beta_{i\cdot},\beta_{l\cdot}\rangle}{\sqrt{\langle\beta_{i\cdot},\beta_{i\cdot}\rangle\langle\beta_{l\cdot},\beta_{l\cdot}\rangle}}\gamma_{ik}\gamma_{lj}\right)s_k s_j$$

where $\langle\beta_{i\cdot},\beta_{l\cdot}\rangle$ is a scalar product of ownership vectors. This measure can not be directly derived from the model discussed in this paper since it is made to be compatible with HHI. Like MHHI, the proposed measure takes a sum of product of market shares with weights. The weights represent an index of incentives alignment: each pair of investors from both firms has weight proportional to their voting rights multiplied by the factor responsible for their incentive alignment. The formula without the incentive alignment factor transforms into $\sum_{k=1}^{N}\sum_{j=1}^{N}\left(\sum_{i=1}^{I}\sum_{l=1}^{I}\gamma_{ik}\gamma_{lj}\right)s_k s_j = 1$ regardless of ownership and voting structures. To understand the motivation for incentive alignment factor recall the first order condition for an investor i at firm k:

$$\beta_{ki}\left(v_k + \sum_{l\neq k}a_{kl}q_l + 2a_{kk}q_k\right) + \sum_{l\neq k}\beta_{li}a_{lk}q_l = 0.$$

This equation is then solved for q_k by investor i as a function of other firms' outputs q_l and parameters a_{kl}, v_k, and investor i shares β_{ki}. By looking at this equation we can see that vector $(\beta_{1i}, ..., \beta_{ki}, ..., \beta_{Ni})$ is orthogonal to the hyperplane which contains "the solution" vector

$$\left(a_{1k}q_1, a_{2k}q_2, ..., a_{k-1,k}q_{k-1}, v_k + \sum_{l\neq k}a_{kl}q_l + 2a_{kk}q_k, a_{k+1,k}q_{k+1}, ..., a_{Nk}q_N\right).$$

Then the incentives alignment factor $\frac{\langle\beta_{i\cdot},\beta_{l\cdot}\rangle}{\sqrt{\langle\beta_{i\cdot},\beta_{i\cdot}\rangle\langle\beta_{l\cdot},\beta_{l\cdot}\rangle}}$ is a cosine of the angle between the hyperplanes for different investors. Recall that each investor solves for q_k given the values of output in other firms q_1, ..., q_N, so the solution is an intersection of vector $(a_{1k}q_1, a_{2k}q_2, ..., a_{k-1,k}q_{k-1}, x, a_{k+1,k}q_{k+1}, ..., a_{Nk}q_N)$ with the hyperplane (where x is determined by the intersection). The angle between the hyperplanes is effectively proportional to the distance between the intersection points' for different investors and may serve as a measure of disagreement between them given other parameters fixed.

We understand that this mathematical reasoning has little to do with economics of the model and may not be considered a valid proof behind an ownership concentration measure. Instead, we would like to think about this as a motivation for an abstract measure, validity of which can be established empirically with actual data and/or numerical simulation. The advantages of this measure are: first, this measure coincides with HHI in case without partial common ownership (as MHHI does); second, it aggregates investors with identical portfolios; third, it uses the same data as MHHI.

5 PHHI and MHHI relationship in data

To back up the claim that PHHI may serve as a concentration measure in industries with common ownership we would like to compare its performance to well known MHHI measure. Of course, this comparison can not be a proof that PHHI is a good concentration measure due to controversial nature of MHHI itself. The goal of this exercise is to show that if there is a significant relationship between PHHI and MHHI then we have a signal that MHHI can be replaced with PHHI (hence some flaws of MHHI can be avoided). Since both MHHI and PHHI are built around HHI the direct comparison between these is not reasonable due to spurious regression problem. Instead, following the existing literature (Azar et al. (2017); Anton et al. (2016); Azar (2017)), we will use a difference between $MHHI$ and HHI. This difference shows the input from common ownership excluding the input from the market shares measured by HHI. It helps to isolate the effect coming from the common ownership. Hence we will consider the relationship between $\Delta MHHI = MHHI - HHI$ and $\Delta PHHI = PHHI - HHI$ to avoid spurious regression problem and to focus only on common ownership components of PHHI and MHHI.

5.1 Data

To create the set of industries for further computation of concentration indices we constructed a set of NAICS codes for the following sectors: Manufacturing, Retail trade, Information, Finance and Insurance, Real Estate and Rental and Leasing. The sample contains 106 different industries defined by the 4-digit NAICS codes. For each such NAICS code in this set, we draw a list of US companies within that code with operating revenue over $100,000. Then for every company from our list we draw a list of current shareholders with unique Orbis (BvD ID) identifiers. If such list is unavailable we make an assumption that this firm maximizes its profit for a unique single investor. Since information on the voting power of shares is not present in this dataset we employ the assumption that each share has one vote. In total, we have 3480 companies in 106 industries and 22454 unique investors in our dataset.

5.2 Results

For every industry in our dataset, we evaluate HHI, MHHI and PHHI indices. Our goal is to see whether there is a statistically significant relationship between $\Delta MHHI$ and $\Delta PHHI$. We do not look for causal relationship since both MHHI and PHHI are mathematical constructs and they can not affect each other. First, we consider a simple regression of $\Delta MHHI$ on $\Delta PHHI$ and a constant term with heteroskedasticity robust standard errors.

$$\begin{array}{rcc} \Delta MHHI = 0.045 & + 0.668\ \Delta PHHI \\ Std.error \quad (0.005) & (0.086) \end{array}$$

The results suggest that there is a statistically significant relationship between $\Delta MHHI$ and $\Delta PHHI$. Moreover, the sign of coefficient is positive as we expected to see. Second, we would like to control for number of firms, number of investors, and initial level of concentration as measured by HHI. This specification yields the following result.

$$\begin{array}{lllllll}\Delta MHHI = & -0.006 + & 0.689\ \Delta PHHI - & 0.001\ N + & 0.0003\ I + & 0.078\ HHI \\ Std.error & (0.009) & (0.077) & (0.000) & (0.0000) & (0.024)\end{array}$$

The coefficient for $\Delta PHHI$ remains significant in this specification as well. This suggests that relationship between these measures is likely to be present across industries with a different number of firms and different number of investors. Since $\Delta MHHI$ is successfully used in the emerging literature, significant relationship between it and $\Delta PHHI$ signals that the latter can be used as a substitute for $\Delta MHHI$. We see the benefits of this replacement in a new possible interpretation of industry concentration measure (as a measure of incentives alignment) and in better treatment of investors with similar portfolios.

Of course, the presented regressions should not be interpreted as an evidence that PHHI performs better or the same as MHHI. The intent of presenting these is to spur attention to the new possible measure of concentration that alleviates some drawbacks of existing measure (MHHI). The goal of this result is to motivate further research on this measure.

6 Conclusion

In this paper, we present a model of firms competition under partial common ownership. We do not assume firms to be profit maximizers, instead we study the relationships between firms' shareholders. Disagreement between the shareholders is solved by an introduction of a simple voting model. Given certain assumptions on shareholders' voting behavior, firms' externalities, and objectives of firms' managers we show an existence of equilibrium.

The equilibrium we study has some distinctive properties. First, the objective of a firm is in the aggregation of shareholders' preferences, as a contrary to simple profit maximization or maximization of a weighted sum of profits of investors' portfolios as existing MHHI-related literature assumes. Second, equilibrium outputs are determined by the median-weighted shareholder in each firm, and the prices are given by the Cournot model with differentiated goods. Third, we show that under specific conditions this equilibrium is unique for a given ownership structure. Fourth, we provide a clear path for finding the equilibrium numerically. A direct solution may not always be feasible due to a significant computational burden. We find that this problem can be alleviated by using a step-by-step convergence approach, and we show specific conditions that guarantee the applicability of it. Fifth, the model effectively aggregates investors with the same portfolios (up to a scale factor) due to perfect alignment of their incentives to vote. This property raises concerns regarding the existing measures of ownership concentration. For instance, MHHI does not have this property. We were able to demonstrate that this may help merging parties to reduce the change in the MHHI to comply with the DOJ/FTC guidelines while keeping the same equilibrium outcome. We propose an alternative measure of ownership concentration, PHHI, that aggregates investors with the same portfolios. Our measure corresponds to HHI when common ownership is absent (as MHHI does as well). Empirically we confirmed that PHHI and MHHI have a statistically significant relationship. This is a signal that PHHI could be a good measure of ownership concentration, but more research is needed to show it.

References

Anton, M., F. Ederer, M. Gine, & M. C. Schmalz (2016) Common Ownership, Competition, and Top Management Incentives. *Working paper*.

Arrow, K. J. & F. Hahn (1971) General competitive analysis. Number xii, 452 p. in Mathematical economics texts,6. San Francisco: Holden-Day.

Azar, J. (2017) Portfolio Diversification, Market Power, and the Theory of the Firm. *Working paper*.

Azar, J., M. C. Schmalz, & I. Tecu (2017) Anti-competitive effects of common ownership. *Ross School of Business Working Paper*.

Benninga, S. & E. Muller (1979) Majority Choice and the Objective Function of the Firm under Uncertainty. *The Bell Journal of Economics* **10**, 670.

Bikker, J. A. & K. Haaf (2002) Measures of competition and concentration in the banking industry: a review of the literature. *Economic & Financial Modelling* **9**, 53–98.

Blackorby, C., D. Donaldson, & J. A. Weymark (1982) A normative approach to industrial-performance evaluation and concentration indices. *European Economic Review* **19**, 89–121.

Bresnahan, T. F. & S. C. Salop (1986) Quantifying the competitive effects of production joint ventures. *International Journal of Industrial Organization* **4**, 155–175.

Davis, G. F. (2008) A new finance capitalism? Mutual funds and ownership re-concentration in the United States. *European Management Review* **5**, 11–21.

DeMarzo, P. M. (1993) Majority Voting and Corporate Control: The Rule of the Dominant Shareholder. *The Review of Economic Studies* **60**, 713.

Downs, A. (1957) An economic theory of political action in a democracy. *The journal of political economy*, 135–150.

Geroski, P. A. (1983) Some reflections on the theory and application of concentration indices. *International Journal of Industrial Organization* **1**, 79–94.

Hansen, R. G. & J. R. Lott (1996) Externalities and Corporate Objectives in a World with Diversified Shareholder/Consumers. *The Journal of Financial and Quantitative Analysis* **31**, 43.

Hart, O. D. (1977) Take-over bids and stock market equilibrium. *Journal of Economic Theory* **16**, 53–83.

Milne, F. (1974) Corporate investment and finance theory in competitive equilibrium. *Economic Record* **50**, 511–533.

Moore, J. C. (1999) *Mathematical Methods for Economic Theory 2*Volume 10 of *Studies in Economic Theory*. Berlin, Heidelberg: Springer Berlin Heidelberg. DOI: 10.1007/978-3-662-08544-8.

Salop, S. & D. O'Brien (2000) Competitive Effects of Partial Ownership: Financial Interest and Corporate Control. *Antitrust L.J.*, 559–614.

Appendix

Equilibrium existence

To show existence of equilibrium in the presented model we are going to use one of the fixed-point theorems and adopt assumptions that are required by it. Consider the Kakutani's fixed-point theorem.

Theorem *(Kakutani) Let $\Gamma : S \to S$ be an upper semi-continuous correspondence from a non-empty, compact, convex set $S \subset \mathbb{R}^n$ into itself such that for all $x \in S$, the set $\Gamma(x)$ is convex and non-empty, then Γ has a fixed point, i.e. there is an x^* where $x^* \in \Gamma(x^*)$.*

We want to extend this theorem to a case where the correspondence arises in a sequence of steps which are related by their own correspondences. Suppose that there are other sets S_1, S_2, ..., S_k and correspondences g_1, g_2, ..., g_k, g s.t. $g_1 : S \to S_1$, $g_2 : S_1 \to S_2$, ..., $g_k : S_{k-1} \to S_k$, $g : S_k \to S$. As long as the composition of the functions $g \circ g_k \circ ... \circ g_1$ is upper semi-continuous, non-empty and convex valued we have at least one fixed point by the theorem above.

Continuity of optimal choices of investors

The first order condition for an investor in combination with irreversibility of production assumption and restriction on upper production limit for a firm gives the following formula for the optimal choice for an arbitrary firm k from the perspective of investor i:

$$b_{ki} = \begin{cases} \max\left[\min\left[\frac{1}{2a_{kk}}\left(-v_k - \sum_{l\neq k} a_{kl}q_l - \frac{1}{\beta_{ik}}\sum_{l\neq k}\beta_{il}a_{lk}q_l\right), Q\right], 0\right] & \text{for} \quad \beta_{ik} > 0 \\ 0 & \text{for} \quad \beta_{ik} = 0 \end{cases}$$

This formula is continuous with respect to the input variables $q_1, ..., q_N$ (firm decisions). The $I \times 1$ vector $b_k = [b_{k1}, ..., b_{kI}]^T$ of optimal views of all investors for a firm k is also continuous wrt the vector of firm's decisions. Then

$B(q_1, ..., q_N) = [b_1^T, ..., b_N^T]^T$ is $N \times I$ matrix of optimal views for all investors and all firms. $B(q_1, ..., q_N)$ maps a compact convex set $X \in \mathbb{R}^N$ of output decisions of firms into a compact convex set $Y \in \mathbb{R}^{NI}$ of optimal views of investors.

Upper hemi-continuity of manager's best response

The manager of a firm n has the objective to maximize his payoff by choosing action g_n in response to the action d_n of contender. Given his payoff function

$$\mathbb{I}\left[\sum_{i=1}^{I} \mathbb{I}[g_n \succsim d_n]\beta_{in} \geq 0.5\right]$$

it is easy to see that his best response has to attract at least 50% of the votes of shareholders. Moreover in case of a tie (i.e. when $g_n \sim d_n$) between the policies shareholders vote for the incumbent manager. Hence the manager of a firm can propose the median-weighted policy which will always be supported by at least 50% of the shareholders. This is due to the fact that shareholders conditional utility function is single-peaked and the first order condition linearly depends on the choice variable, hence each shareholder will vote for the proposed policy that is close to his optimal view.

Suppose b_{ki} is the optimal view of shareholder i on firm's k production decision and $b_k = [b_{k1}, ..., b_{kI}]^T$ is the vector of views of all shareholders. Then the best response correspondence of manager of the firm is

$$g_k(b_k) = \left\{ g \in [0, Q] : \mathbb{I}\left[\sum_{i=1}^{I} \mathbb{I}[g \geq b_{ki}]\beta_{ik} \geq 0.5\right] \mathbb{I}\left[\sum_{i=1}^{I} \mathbb{I}[g \leq b_{ki}]\beta_{ik} \geq 0.5\right] > 0 \right\}$$

which is non-empty upper hemi-continuous convex valued correspondence. Now we can construct the best-response correspondence of all incumbent managers together (we do not include the best responses of contenders here since these are $[0, Q]$ in equilibrium):

$$G(b_1, ..., b_N) = [g_1(b_1), ..., g_N(b_N)]^T$$

which is also non-empty upper hemi-continuous and convex valued with codomain $[0, Q]^N$. Note that $G(b_1, ..., b_N)$ maps the compact convex set $Y \in \mathbb{R}^{NI}$ of optimal views of investors into a compact convex set $X \in \mathbb{R}^N$ of output decisions of firms.

Since $B(q_1, ..., q_N)$ is a continuous correspondence from $\mathbb{R}^N \to \mathbb{R}^{NI}$ and $G(b_1, ..., b_N)$ is an upper hemi-continuous correspondence from $\mathbb{R}^{NI} \to \mathbb{R}^N$ the composition $G \circ B$ is an upper hemi-continuous correspondence as well by Berge theorem (Moore (1999)). This completes the proof of equilibrium existence since now all the requirements of Kakutani theorem are satisfied.

Note that here we do not involve the best response functions of contenders. The reason to do so is that in equilibrium every firm's manager retains his place and contender does not get that place, so his best response correspondence in equilibrium is any possible choice $\{g \in [0, Q]\}$. For simplicity, we then assume that since his best response covers the entire set we can say that contender always proposes firm's policy according to the same best response correspondence $g_k(b_k)$ as incumbent manager does.